# Solution for the integral $\int \frac{dx}{\sin^3(2x)}$.

I was asked about the integral

$$\int \frac{dx}{\sin^3(2x)},$$

here is the way to do it: The trick (or at least, *one* trick), is to use the trigo equality

$$\sin(2x) = 2\sin(x)\cos(x) :$$

That way, the integral becomes:

$$\frac{1}{8}\int \frac{dx}{\sin^3(x)\cos^3(x)}.$$

Note: we have cos's, this remind of sec. Thus let's try to change sin for a tan:

$$\sin(x) = \tan(x)\cos(x) :$$

$$\frac{1}{8}\int \frac{dx}{\sin^3(x)\cos^3(x)} = \frac{1}{8}\int \frac{dx}{\tan^3(x)\cos^6(x)} = \frac{1}{8}\int \frac{\sec^6(x)dx}{\tan^3(x)}.$$

Note that we know that $\sec^2(x) = 1+\tan^2(x)$. Thus this is calling for the substitution $u = \tan(x)$: then $du = \sec^2(x)dx$:

$$\frac{1}{8}\int \frac{\sec^6(x)dx}{\tan^3(x)} = \frac{1}{8}\int \frac{(\sec^2(x))^2\sec^2(x)dx}{\tan^3(x)} = \frac{1}{8}\int \frac{(\sec^2(x))^2du}{u^3} = \frac{1}{8}\int \frac{(1+\tan^2(x))^2du}{u^3}$$

$$= \frac{1}{8}\int \frac{(1+u^2)^2du}{u^3} = \frac{1}{8}\int \frac{(1+2u^2+u^4)du}{u^3} = \frac{1}{8}\int (\frac{1}{u^3} + \frac{2}{u} + u)du = \frac{u^2}{2} + 2\ln|u| - \frac{1}{2u^2}$$

Since $u = \tan(x)$,

$$\int \frac{dx}{\sin^3(2x)} = \frac{\tan(x)^2}{2} + 2\ln|\tan(x)| - \frac{1}{2\tan(x)^2}$$