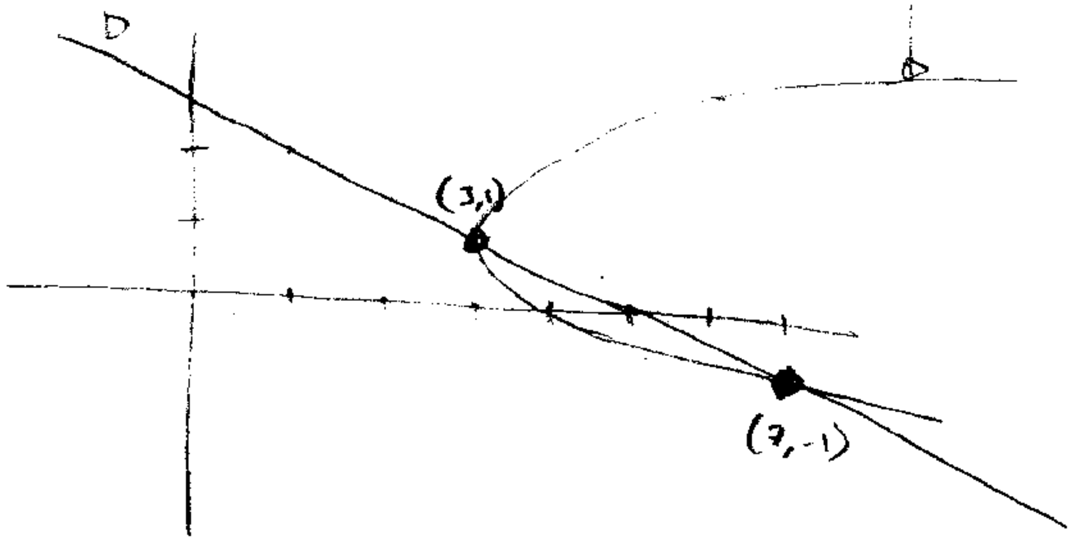
(5) region $\begin{cases} x-3=(y-1)^2 \\ x=5-2y \end{cases}$



intersection pts: $(y-1)^2+3=5-2y$

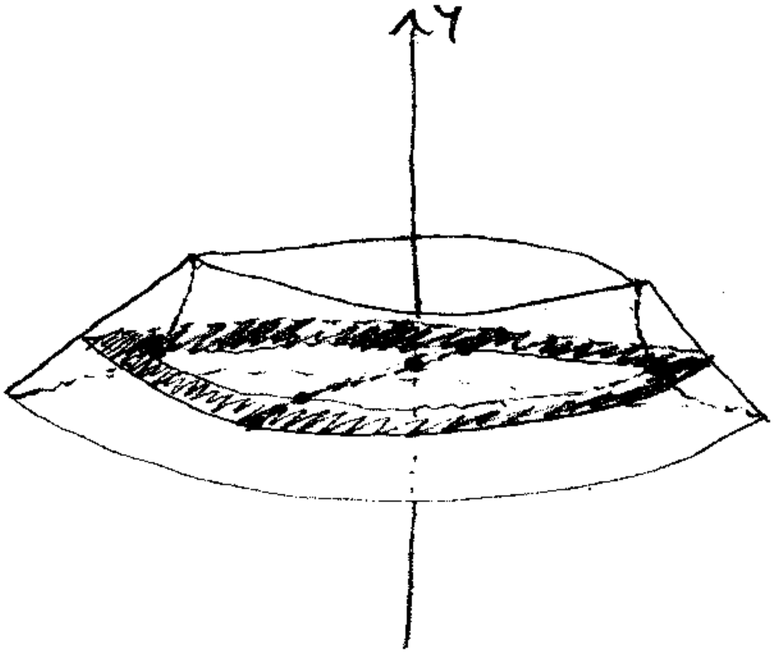
$y^2-2y+1+3=5-y$

$y^2-1=0$

$y=1$ or $y=-1$

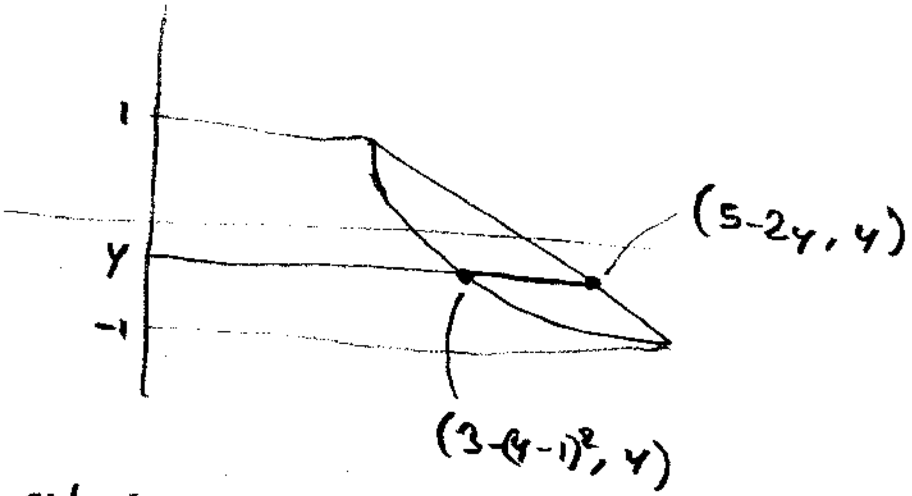
$\rightarrow (3,1)$ and $(7,-1)$

(2) rotation around the y-axis:



a cross-section perp. in the region gives an annulus:

and if the cross-section is at height y:



the exterior circle of the annulus has $5-2y$ for radius
interior one (the hole) has $3-(y-1)^2$ for radius.
Thus the area of the annulus is

$\pi R_{ext}^2 - \pi R_{int}^2$

$= \pi(5-2y)^2 - \pi(3-(y-1)^2)^2$

$= \pi((5-2y)^2 - (3-(y-1)^2)^2)$

Thus the volume of an annulus of thickness dy is

$\pi((5-2y)^2 - (3-(y-1)^2)^2)\,dy$

and the volume of the solid of revolution is

$$\pi\int_{-1}^{1} ((5-2y)^2 - (3-(y-1)^2)^2)\,dy$$

$$= \pi\underbrace{\int_{-1}^{1}(2y-5)^2\,dy} - \pi\underbrace{\int_{-1}^{1}(3-(y-1)^2)^2\,dy}$$

$t = 2y-5$
$dt = 2\,dy$
$y=1 \Rightarrow t=-3$
$y=-1 \Rightarrow t=-7$

$t = y-1$
$dt = dy$
$y=1 \to t=0$
$y=-1 \to t=-2$

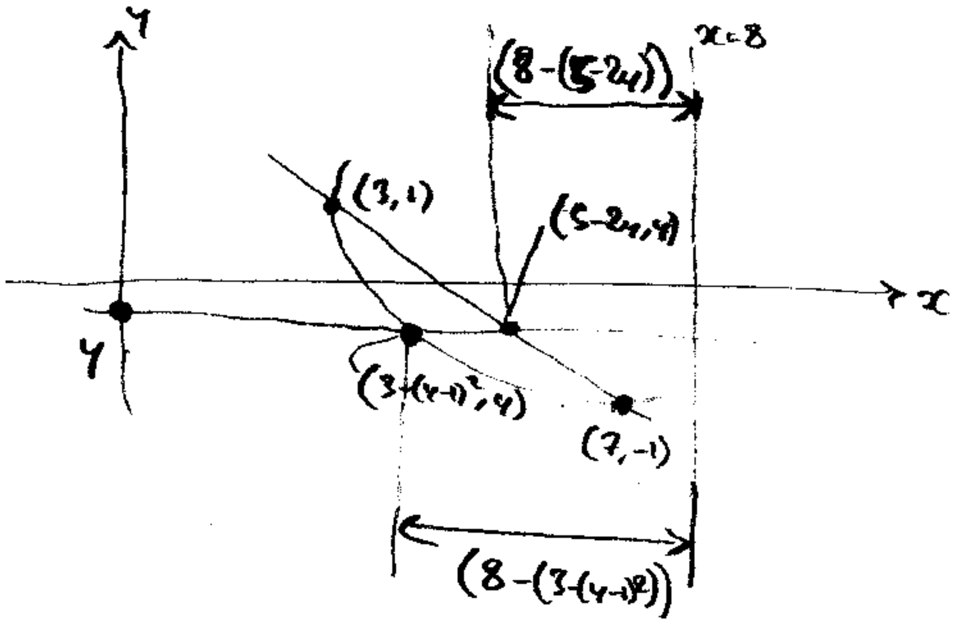
$$= \pi\int_{-7}^{-3}\frac{t^2}{2}\,dt - \pi\int_{-2}^{0}(3-t^2)^2\,dt$$

$$= \pi\left[\frac{t^3}{6}\right]_{-7}^{-3} - \pi\int_{-2}^{0}(9-6t^2+t^4)\,dt$$

$$= \pi\left(\frac{(-3)^3}{6} - \frac{(-7)^3}{6}\right) - \pi\left[9t - \frac{6t^3}{3} + \frac{t^5}{5}\right]_{-2}^{0}$$

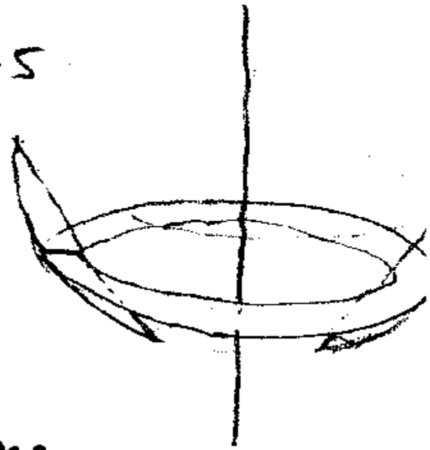
$$= \frac{158}{3}\pi - \frac{42\pi}{5} = \frac{664}{15}\pi$$

(ii) rotated around $x = 8$:



here, $R_{int} = 8-(5-2y) = 2y+3$

$R_{ext} = 8-(3-(y-1)^2) = (y-1)^2+5$



→ annulus has an area of

$$\pi\left(((y+1)^2+5)^2-(2y+3)^2\right)$$

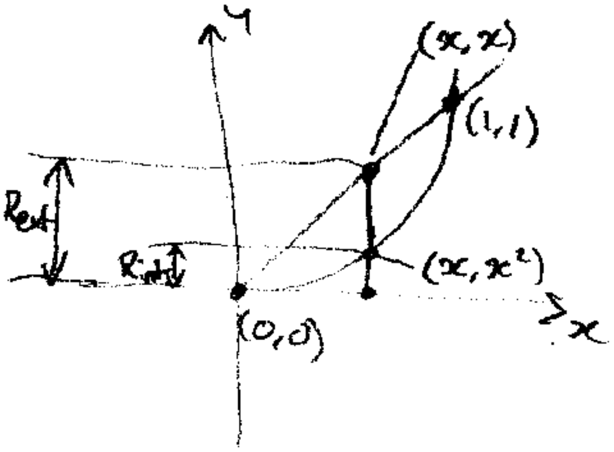
→ the volume generated has volume

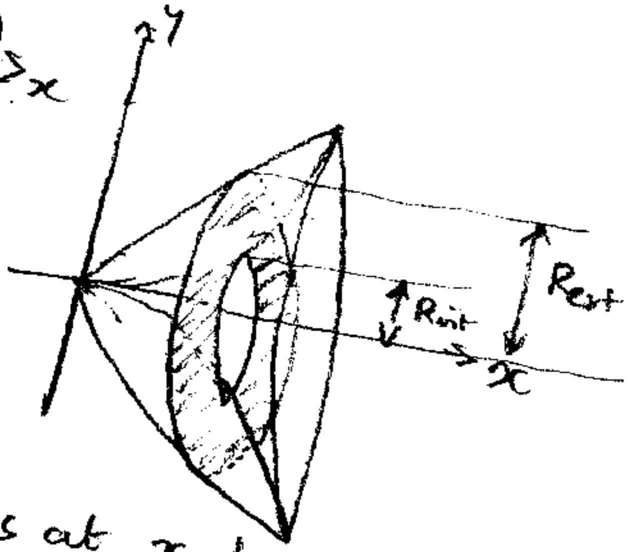
$$\pi\int_{-1}^{1}((y+1)^2+5)^2-(2y+3)^2\,dy$$

$$= \cdots = \frac{312}{5}$$

⑥ region: $\begin{cases} y = x^2 \\ y = x \end{cases}$

(i) around the x-axis:





The annulus at x has
internal radius $R_{int} = x^2$
ext radius $R_{ext} = x$

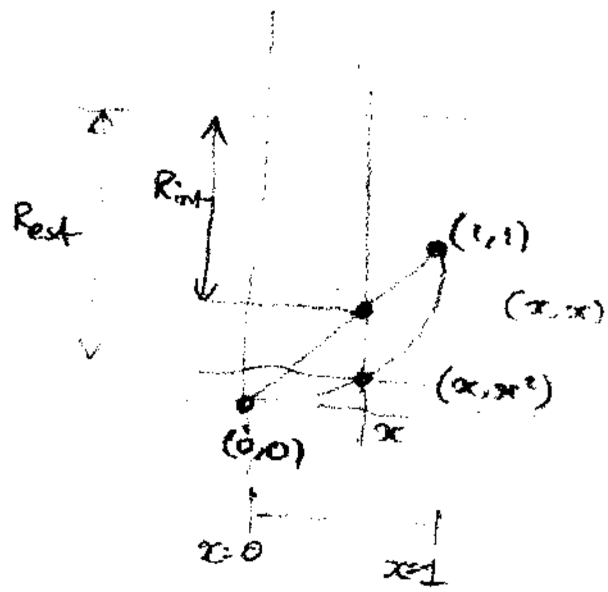
$\rightarrow$ area of the annulus is
$\pi(x)^2 - \pi(x^2)^2 = \pi(x^2 - x^4)$

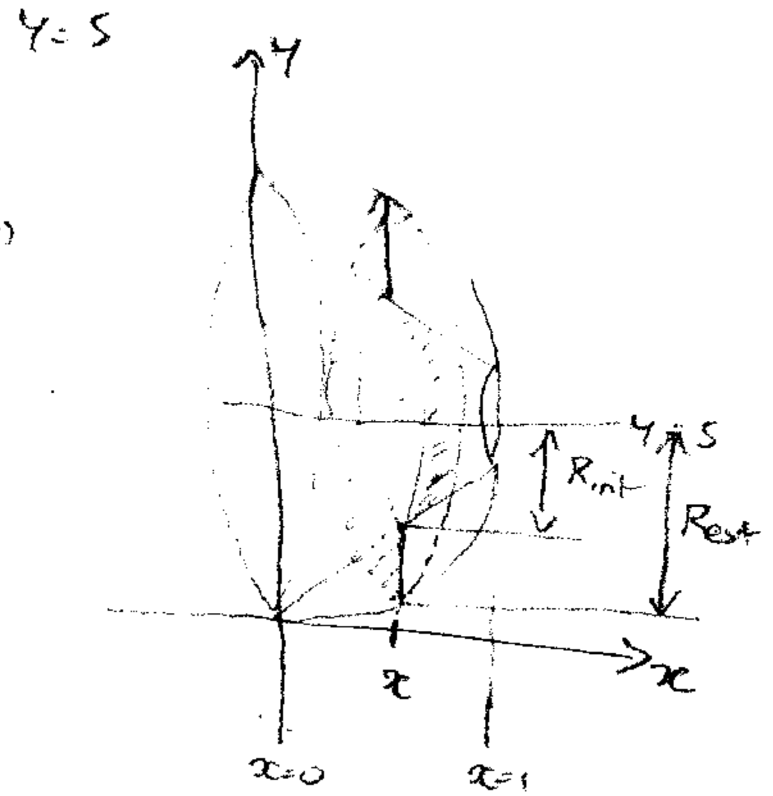
$\rightarrow$ volume of revolution is

x goes from 0 to 1

$$\int_0^1 \pi(x^2 - x^4)\,dx = \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$
$$= \pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}.$$

(ii) around $y = 5$:





$\rightarrow$

$R_{ext} = 5 - x^2$

$R_{int} = 5 - x$.

So area of annulus is

$$\pi(R_{ext}^2 - R_{int}^2)$$

$$= \pi((5-x^2)^2 - (5-x)^2)$$

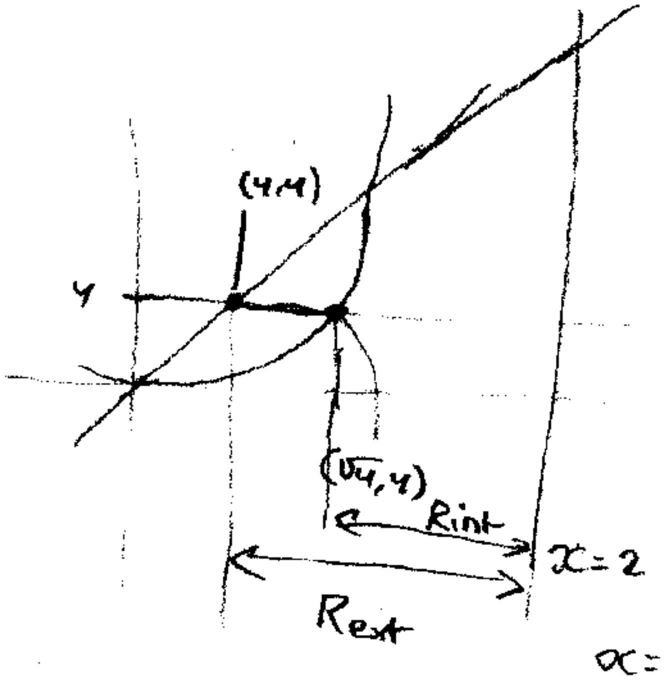
$$= \pi(\cancel{25} - 10x^2 + x^4 - \cancel{25} + 10x - x^2)$$

$$= \pi(x^4 - 11x^2 + 10x)$$

$\rightarrow$ total volume is

$$\pi\int_0^1 (x^4 - 11x^2 + 10x)\,dx = \frac{23\pi}{15}$$

(iii) the line $x=2$

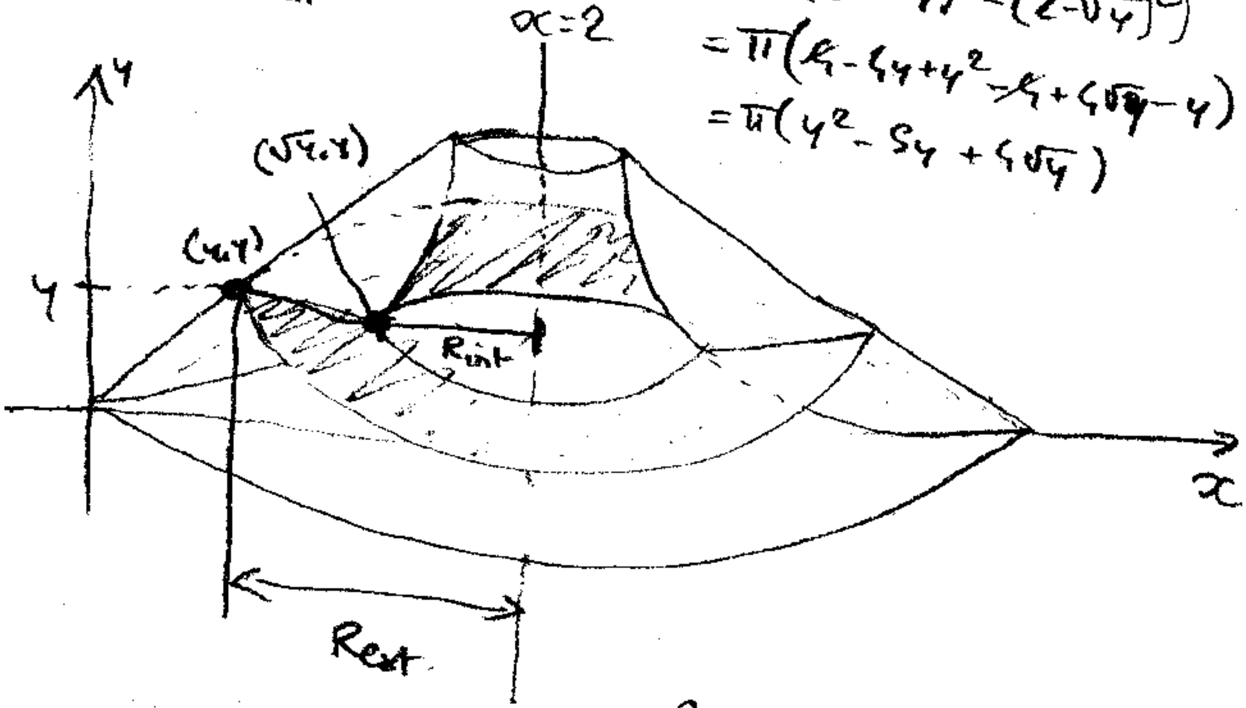


$R_{ext} = 2-y$

$R_{int} = (2-\sqrt{y})$

$\rightarrow$ area of the annulus is

$$\pi\left((2-y)^2-(2-\sqrt{y})^2\right)$$
$$=\pi\left(\cancel{4}-4y+y^2-\cancel{4}+4\sqrt{y}-y\right)$$
$$=\pi\left(y^2-5y+4\sqrt{y}\right)$$



So volume total is $\pi\int(y^2-5y+4\sqrt{y})\,dy$

$$=\frac{\pi}{2}$$