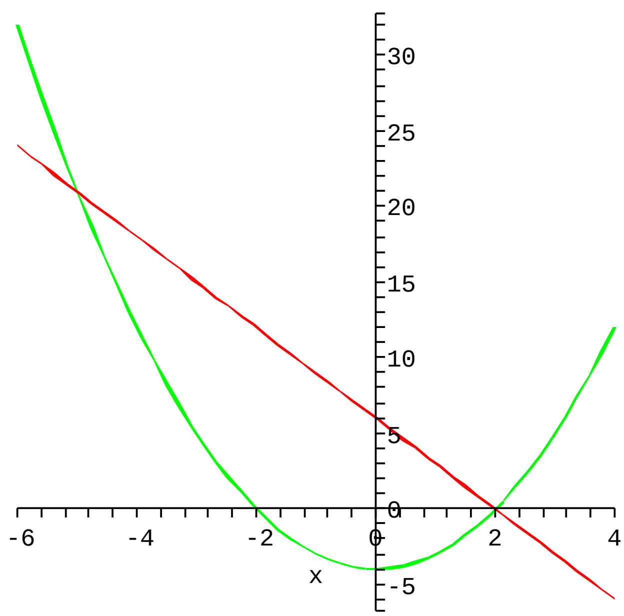
Curves, and partial solutions for the list of suggested exercises,
Sections 8.1-2 (and a bit of 5.2)
For the DGD of May 22th and 24th

(1) Find the area between the following curves:

$$(i) \quad \underset{\text{(red)}}{3x + y = 6} \quad \underset{\text{(green)}}{y = x^2 - 4}$$

The curves looks like:

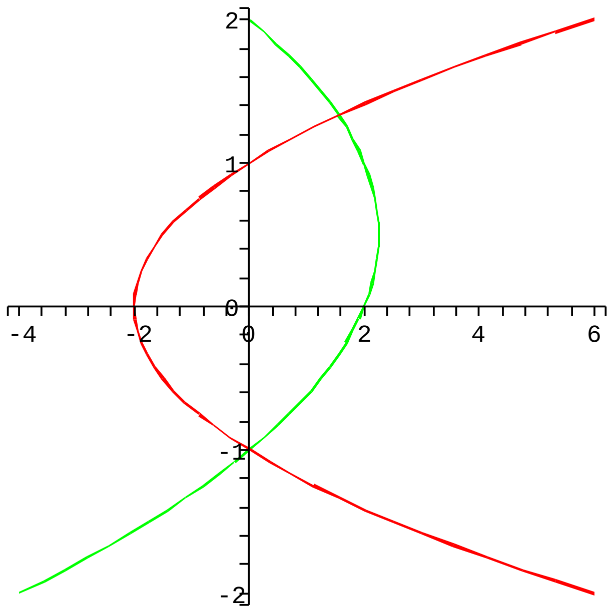


The intersection points are at $x = -5, 2$. The area is then:

$$\int_{-5}^{2} (6 - 3x) - (x^2 - 4)dx = \frac{343}{6}$$

$$(ii) \quad \underset{\text{(red)}}{2y^2 = x + 2} \quad \underset{\text{(green)}}{y^2 + x = y + 2.}$$

The curves looks like:

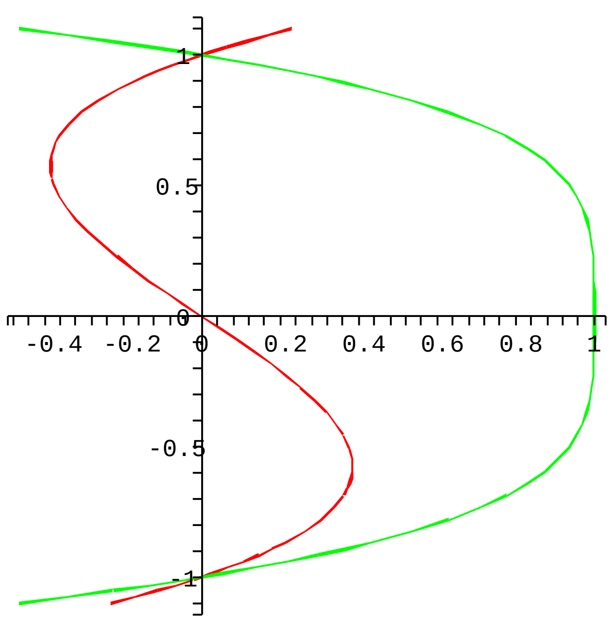


The intersection points are at $x = -1, \frac{4}{3}$. The area is then:

$$\int_{-1}^{\frac{4}{3}} (y - y^2 + 2) - (2y^2 - 2)dy = \frac{343}{54}$$

$$\begin{array}{lll} (iii) & x = y^3 - y & x = 1 - y^4. \\ & \text{(red)} & \text{(green)} ** \end{array}$$

The curves looks like:

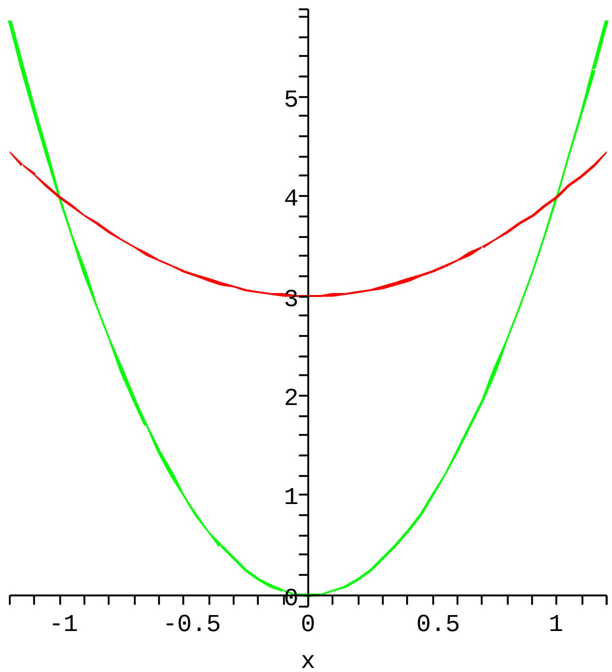


The intersection points are at $x = -1, 1$. The area is then:

$$\int_{-1}^{1} (1 - y^4) - (y^3 - y)dy = \frac{8}{5}$$

$$(iv) \quad \underset{\text{(green)}}{y = 4x^2} \quad \underset{\text{(red)}}{y = x^2 + 3}$$

The curves looks like:

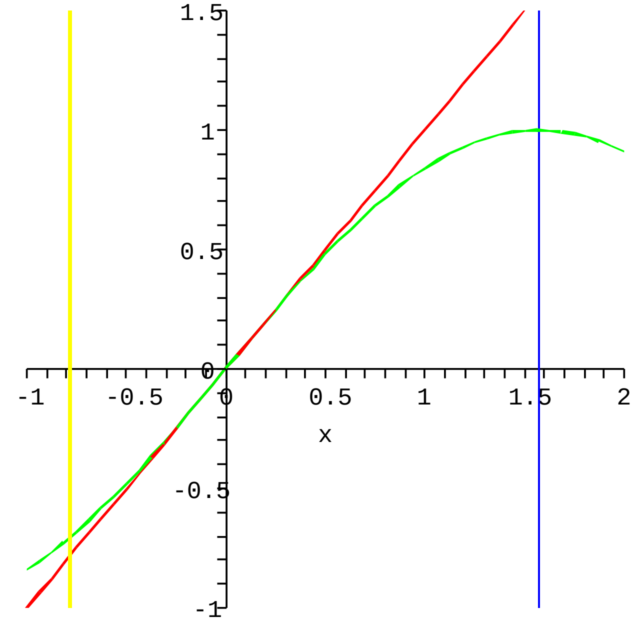


The intersection points are at $x = -1, 1$. The area is then:

$$\int_{-1}^{1} (x^2 + 3) - (4x^2)dx = 4$$

$$(v) \quad \underset{\text{(green)}}{y = x} \quad \underset{\text{(red)}}{y = \sin(x)} \quad \underset{\text{(yellow)}}{x = -\tfrac{\pi}{4}} \quad \underset{\text{(blue)}}{x = \tfrac{\pi}{2}}.$$

The curves looks like:



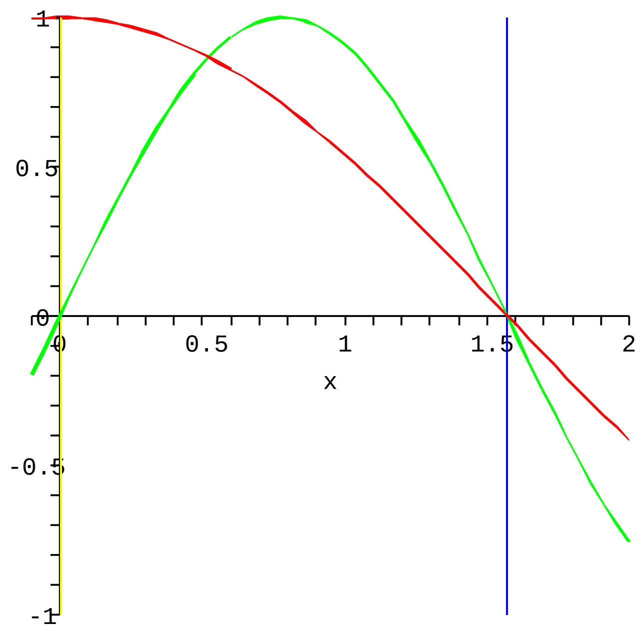
The intersection point is 0. Before 0, $\sin(x)$ is above x, after it is below. The area is then:

$$\int_{-\frac{\pi}{4}}^{0} (sin(x) - x)dx + \int_{0}^{\frac{\pi}{2}} (x - \sin(x))dx = \frac{\sqrt{2}}{2} + \frac{5\pi^2}{32} - 2$$

Note that there is a pdf containing the full computation.

(vi)	$y = \cos(x)$	$y = \sin(2x)$	$x = 0$	$x = \frac{\pi}{2}$.
	(red)	(green)	(yellow)	(blue)

The curves looks like:

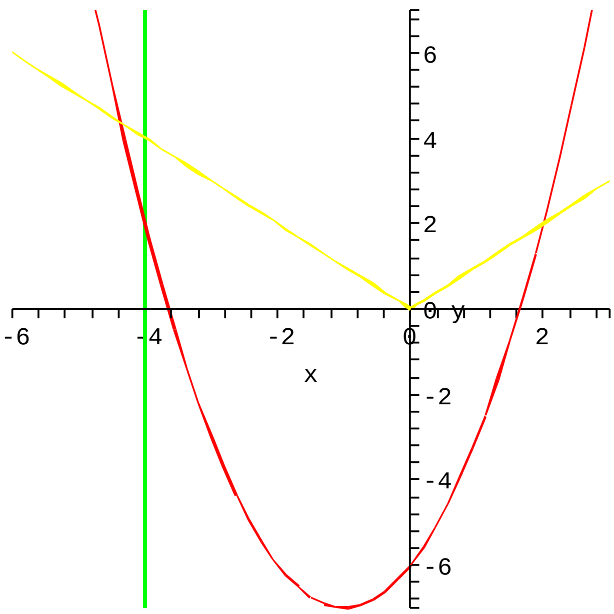


The intersection points is $\pi/6$. Before $\pi/6$, $\cos(x)$ is above $\sin(x)$, after it is below. The area is then:

$$\int_0^{\frac{\pi}{6}} (\cos(x) - \sin(x))dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin(x) - \cos(x))dx = \frac{1}{2}$$

$$(vii) \quad \underset{\text{(yellow)}}{y = |x|} \quad \underset{\text{(red)}}{y = (x+1)^2 - 7} \quad \underset{\text{(green)}}{x = -4.}$$

The curves looks like:



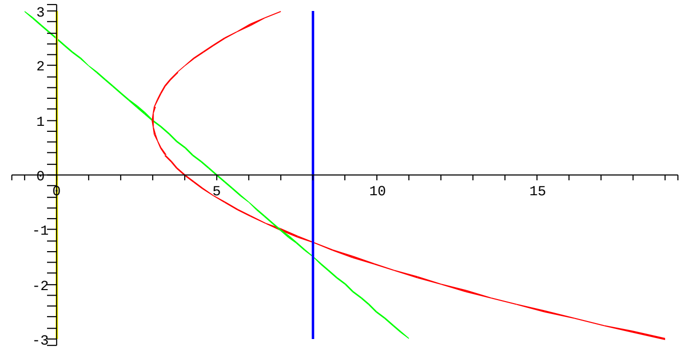
The intersection points are 2 and some number smaller than -4. When $x \leq 0$, $|x| = -x$ and when $x \geq 0$, $|x| = x$. The area is then:

$$\int_{-4}^{0} (-x - ((x+1)^2 - 7))dx + \int_0^2 (x - ((x+1)^2 - 7))dx = 34$$

For (5),(6) and (8) have a look at the two other pdf's: they contain detailed solutions

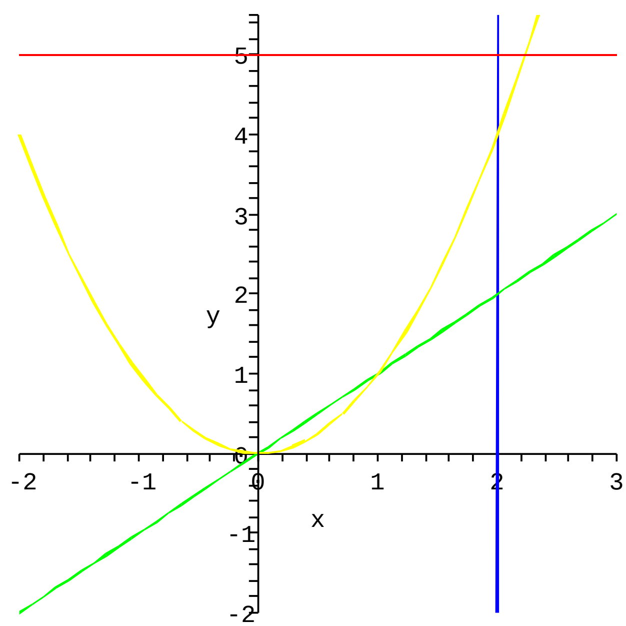
(5) Consider the region delimited by $x - 3 = (y-1)^2$ (red) and $x = 5 - 2y$ (green). Compute the volume of the solid generated by rotating the region around:

1. the y-axis;
2. the line $x = 8$ (blue).



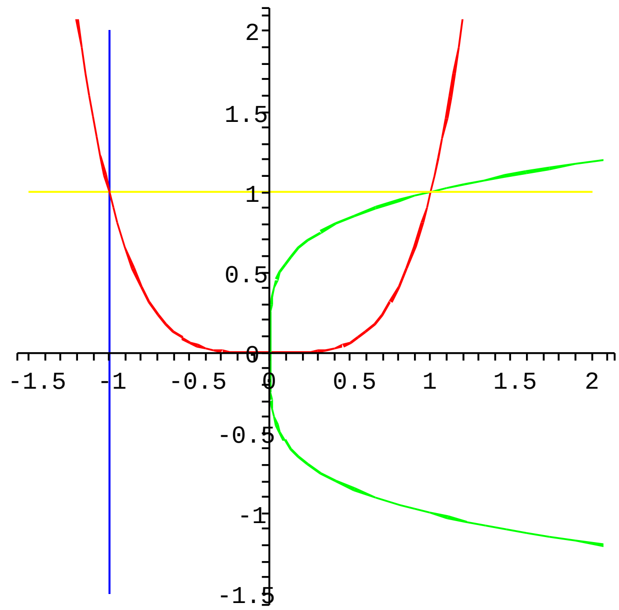
(6) Consider the region bounded by the curves $y = x$ (green) and $y = x^2$ (yellow). Find the volume of the solid generated by rotating the region around

1. the x-axis;
2. the line $y = 5$ (red);
3. the line $x = 2$ (blue).



(7) Consider the region bounded by the curves $y = x^4$ (red) and $x = y^4$ (green). Find the volume of the solid generated by rotating the region around

1. the x-axis;
2. the y-axis;
3. the line $y = 1$ (yellow);
4. the line $x = -1$ (blue).



(8) Consider the region bounded by the curves $y = sin(x)$ (yellow), $x = \pi/2$ (green) and the x-axis. Find the volume of the solid generated by rotating the region around

1. the x-axis;
2. the y-axis;
3. the line $y = 1$ (red);
4. the line $x = 2$ (blue).