# List of suggested exercises, Sections 8.4-5
For the DGD of May 29th and 31st

Note: I indicate the exercise number from the book when available. However, the question asked might be slightly different.

1 Density and center of mass, Section 8.4

(1) (#1) Find the mass of a rod of length 10 cm with density $\delta(x) = e^{-x}$ g/cm at a distance x cm from the left end. What is its center of mass ?

(2) (#10) The density of oil in a circular oil slick on the surface of the ocean at a distance r meters from the center of the slick in given by $\delta(r) = 50/(1+r)$ kg/m^2.

    (a) If the slick extends from $r = 0$ to $r = 10000$ meters, find the exact value of the mass of oil in the slick.

    (b) Within what distance r is half of the oil of the slick contained ?

(3) Consider two functions f and g defined on $[a, b]$, such that $f(x) > g(x)$. A region is bounded on the left by the line $x = a$ and on the right by the line $x = b$, above by $y = f(x)$, below by $y = g(x)$. If x is given in centimeters, and if the density of the region $\delta(x)$ gm/cm^2 varies only in term of x:

    (a) find the total mass of the region,

    (b) find the x-coordinate of the center of mass.

(4) (#13) The storage shed:
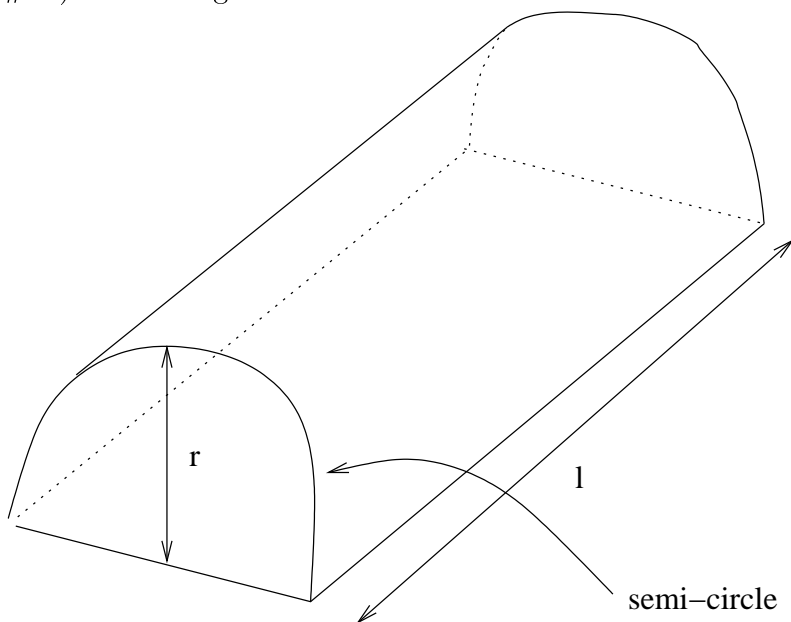
is the shape of a half-cylinder of radius r and length l.

    (a) What is the volume of the shed ?

    (b) The shed is filled with sawdust whose density (mass/unit volume) at any point is proportional to the distance from that point to the floor. The constant of proportionality is k. Calculate the total mass of sawdust in the shed.

(4) (#16) An exponential model for the density of the earth's atmosphere says that if the temperature of the atmosphere were constant, then the density of the atmosphere as a function of height, h (in meter), above the surface of the earth, would be given by
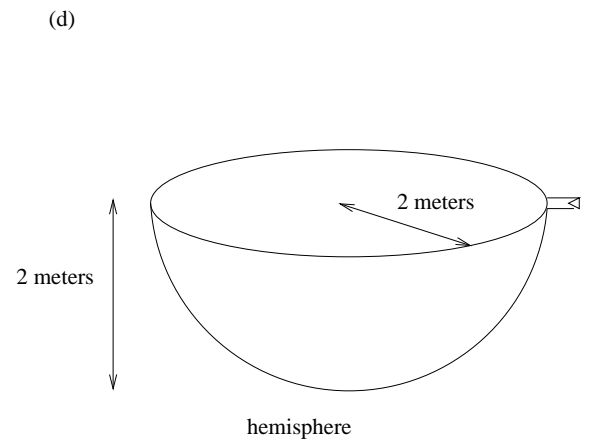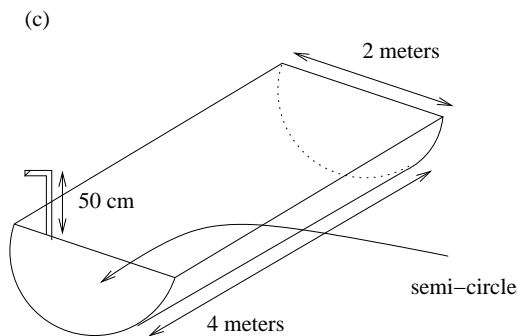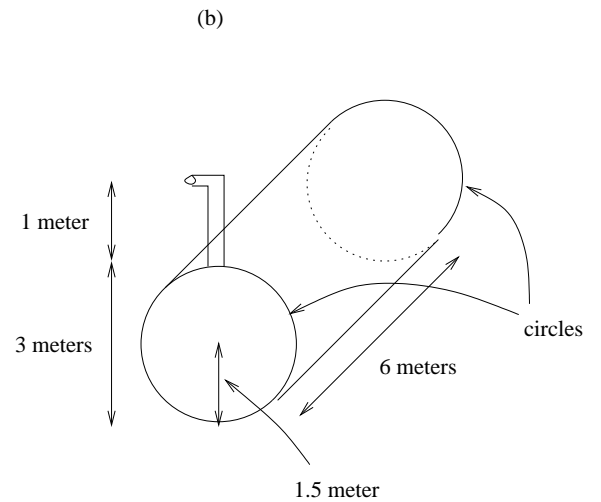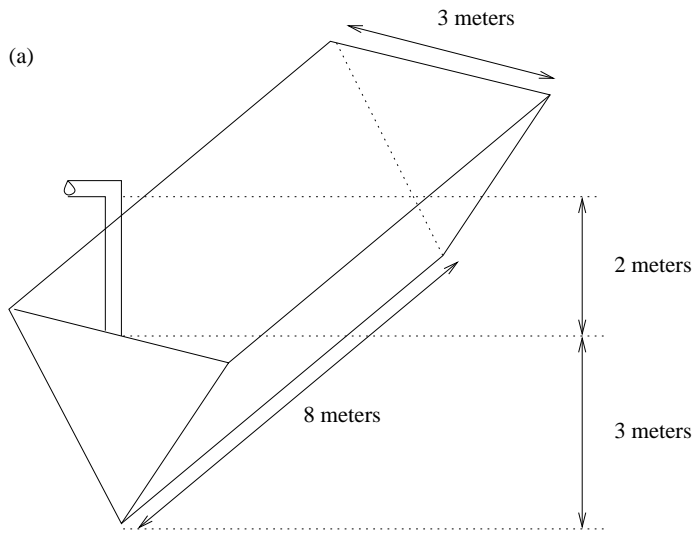
$$\delta(h) = 1.28e^{-0.000124h}\text{kg/m}^3.$$

Find the mass of the portion of the atmosphere from $h = 0$ to $h = 100$ meters (i.e. the first 100 meters above sea-level). Assume the radius of the earth is 6400 km.

(5) (#23) A metal plate, with constant density 2 g/cm^2, has a shape bounded by the curve $y = x^2$ and the x-axis, for $x \in [0, 1]$, x and y in cm.

  (a) Find the total mass of the plate.

  (b) Find the coordinates of the center of mass.

(6) (#24) Same question, but plate is bounded by $y = \sqrt{x}$, the x-axis, $x \in [0, 1]$, and the density of the plate is 5 gm/cm^2.

(7) Find the center of mass of an homogeneous (i.e. uniform density) isosceles triangle of base 5 cm and height 7 cm.

(8) (#26) Find the center of mass of a cone of height 5 cm and round base of diameter 10 cm with constant density δ cm/gm^3.

(9) (#27) A solid is formed by rotating the region bounded by the curve $y = e^{-x}$ and the x-axis between $x = 0$ and $x = 1$ around the x-axis. Assuming the solid has uniform density δ, find the center of mass.

(10) (#28) Find the center of mass of a pyramid of constant density δ gm/cm^3 with a square base of side 40 cm and height 10 cm (that is, the vertex is 10 cm above the center of the base). Find the center of mass of the pyramid.

2 Work, Section 8.5

(1) A bucket of mass 1 kg and a rope of negligible weight are used to draw water from a well that is 25 meters deep. The bucket starts with 20 liters of waters and is pulled up at a rate of 0.5 meter/second, but water leaks out of a hole at a rate of 0.2 liter per second. find the work done in pulling the bucket to the top of the well. (1 liter of water is 1 kg.)

(2) An aquarium is 2 meter long, 1 meter wide, 1 meter deep, and is full of water. Computer the work needed to pump half of the water out of the aquarium.

(3) A tank is full of water. find the work required to pump the water out of the outlet (1 m^3 of water is 1000 kg), in each of the four cases:

(a)

3 meters

8 meters

2 meters

3 meters

(b)

1 meter

3 meters

6 meters

circles

1.5 meter

(c)

2 meters

50 cm

4 meters

semi−circle

(d)

2 meters

2 meters

hemisphere

(**4**) Suppose that for the tank in question (2.a) the pump breaks after $4.7 \cdot 10^5$ Joules of work has been done. What is the depth of the water remaining in the tank ?

(**5**) Solve question (2.b) if the tank is half-full of oil, having density 920 kg/m^3.

(**6**) A cable of density 1 kg/meter is used to lift 400 kg of coal up a mineshaft 150 meters deep. Find the work done.

(**7**) (#5) How much work is required to lift a 1000-kg satellite to an altitude of $2 \cdot 10^6$ meters above the surface of the earth ? The gravitational force is $F = GMm/r^2$, where M is the mass of the earth, m the mass of the satellite, r the distance between their centers. Consider the earth and the satellite as being point-masses. The radius of the earth is $6.4 \cdot 10^6$ meters, its mass is $6 \cdot 10^{24}$ kg, and in these units the gravitational constant G is $6.67 \cdot 10^{-11}$.

(**8**) (#3) The force required to compress a spring by a distance of x meters is given by $F = 3x$ newtons.

   (a) Find the work in compressing the spring from the equilibrium position $x = 0$ to $x = 2$,

   (b) from the equilibrium $x = 4$ to $x = 6$.

(9) A piece of ice is attached to a rope of negligible mass. The ice is meting: its mass in function of time is $m(t) = 10e^{0.1t}$ kg, where t is in minutes. How much work is done when the piece of ice is lifted over 50 meters at a speed of 1 meter per minute ?