example: $\int \frac{x^2+1}{x+1}\,dx$.

$$
\begin{array}{r|l}
x+1 & x^2+1 \\
 & -(x^2+x) \\
\hline
 & -x+1 \\
 & -(-x-1) \\
\hline
 & 2
\end{array}
\qquad x-1
$$

So $x^2+1 = (x+1)(x-1)+2$.

Thus $\frac{x^2+1}{x+1} = \frac{(x+1)(x-1)}{x+1} + \frac{2}{x+1}$

$= x-1 + \frac{2}{x+1}$

Coming back to the integral:

$$\int \frac{x^2+1}{x+1}\,dx = \int \left(x-1+\frac{2}{x+1}\right) dx$$

$$= \frac{1}{2}x^2 - x + 2\ln|x+1| + C.$$

example. $\int \frac{x^2+1}{x-1}\,dx$

$$x-1 \;\overline{)\; x^2+1 \;}\;\; x+1$$
$$-(x^2-x)$$
$$x+1$$
$$-(x-1)$$
$$2$$

So $x^2+1 = (x+1)(x-1)+2$

Thus

$$\frac{x^2+1}{x-1} = \frac{(x+1)(x-1)}{x-1} + \frac{2}{x-1}$$

$$= x+1+\frac{2}{x-1}.$$

and the integral is

$$\int \frac{x^2+1}{x-1}\,dx = \int \left(x+1+\frac{2}{x-1}\right)dx$$

$$= \frac{1}{2}x^2 + x + 2\ln|x-1| + C.$$