example of partial Fractions:

$$\frac{P(x)}{Q(x)} = \frac{5x-2}{(x+1)^3(2x-1)}$$

→ the role tells us that there are A, B, C and D st

$$\frac{P(x)}{Q(x)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{2x-1}.$$

Factorizing, one gets

$$= x^3(2A+D) + x^2(3A+2B+3D) + x(B+2C+3D) + (D-A-B-C).$$

Thus A, B, C and D satisfies

$$\begin{cases} 0 = 2A + D & (1) \\ 0 = 3A + 2B + 3D & (2) \\ 5 = B + 2C + 3D & (3) \\ -2 = -A - B - C + D. & (4) \end{cases}$$

in (1) gives

Doing (1) = (1) + (4) × 2 , $\begin{cases} -4 = \quad -2B - 2C + 3D & (1) \\ -6 = \quad -B - 3C + 6D & (2) \\ 5 = \quad B + 2C + 3D & (3) \\ -2 = -A - B - C + D & (4) \end{cases}$
(2) = (2) + (4) × 3

Doing (1) = (1) + (3) × 2 : $\begin{cases} 6 = \quad 2C + 9D & (1) \\ -1 = \quad -C + 9D & (2) \\ 5 = \quad B + 2C + 3D & (3) \\ -2 = -A - B - C + D & (4). \end{cases}$
(2) = (2) + (3)

Doing (1) = (1) + (2) × 2 : $\begin{cases} 4 = \quad 27D & (1) \\ -1 = \quad -C + 9D & (2) \\ 5 = \quad B + 2C + 3D & (3) \\ -2 = -A - B - C + D & (4). \end{cases}$

Thus $D = \frac{4}{27}$ (From (1))

(2) gives $C = 1 + 9D = 1 + \frac{9 \times 4}{9 \times 3} = 1 + \frac{4}{3} = \frac{3}{3} + \frac{4}{3} = \frac{7}{3}$

(3) gives $B = 5 - 2C - 3D = 5 - \frac{14}{3} - \frac{4}{9} = \frac{45}{9} - \frac{42}{9} - \frac{4}{9}$
$= -1/9$

(4) gives $A = 2 - B - C + D = 2 + \frac{1}{9} - \frac{7}{3} + \frac{4}{27} = \frac{54}{27} + \frac{3}{27} - \frac{63}{27} + \frac{4}{27}$
$= \frac{-2}{27}$.