List of suggested exercises, Functions of several variables, Chapters 12-14.

(1) For each of the following functions:

(a) Compute all the partial derivatives.

(b) Gives the directional derivative at $(1,2)$ along the vector $(2,\sqrt{5})$. Is the function increasing or decreasing along this vector ?

(c) When sitting at $(0,-1)$, in what direction should we go to find the highest rate of change of f ?

$$f(x,y)=e^{-x^2-y^2} \qquad f(x,y)=x^4+(y+1)^2 \qquad f(x,y)=\frac{1}{\sqrt{x^2+y^2+3}}$$

$$f(x,y)=\frac{x+y}{xy-1} \qquad f(x,y)=\frac{x}{x+y^2} \qquad f(x,y)=\sin(x^2)+y$$

(2) For each of the following equation, make sure that the given point is on the surface. Then find a vector normal and the tangent plane to the surface at this point.

$$3=xy+yz+zx \qquad (1,1,1)$$

$$\frac{1}{2}=\sin(xyz) \qquad (\frac{\pi}{3},\frac{1}{2},1)$$

$$4=(x+y)^2z^2+x^3y^3 \qquad (0,1,2)$$

(3) For each of the following function, give a contour diagram and draw a few gradient vectors from points on these curves. From $(1,1)$, where should we go to find the maximum rate of change of the function ?

$$f(x,y)=e^{2-x^2-y^2} \qquad f(x,y)=x^2-(y-1)^2 \qquad f(x,y)=1+x^2+y^2$$

(4) So that you can check you understood how to construct the chain rule:

(a) Given $t=f(x,y,z)$ and $x=g(u,v)$, $y=h(u,v)$, $z=k(u,v)$, can you write the partial derivatives along u and v of $t=f(g(u,v),h(u,v),k(u,v))$?

(b) Same question with $t=f(x,y)$ and $x=g(u,v,w)$, $y=h(u,v,w)$: what are the partial derivatives along u, v and w of the function $t=f(g(u,v,w),h(u,v,w))$?

(5) Compute $\frac{df}{dt}$ using the chain rule when

$$f(x,y)=xy^2 \qquad x=e^{-t} \qquad y=sin(t)$$

$$f(x,y)=xe^y \qquad x=2t \qquad y=1-t^2$$

$$f(x,y)=x\sin(y)+y\sin(x) \qquad x=t^2 \qquad y=\ln(t)$$

(6) Compute $\partial f/\partial u$ and $\partial f/\partial v$ using the chain rule when

$$f(x,y)=\sin\left(\frac{x}{y}\right) \qquad x=\ln(u) \qquad y=v$$

$$f(x,y)=xe^y \qquad x=u^2+v^2 \qquad y=u^2-v^2$$

$$f(x,y)=xe^{-y}+ye^{-x} \qquad x=u\cos(v) \qquad y=v\cos(u)$$

$$f(x,y)=x^2y-y^2 \qquad x=v+u \qquad y=u^2$$

$$f(x,y)=(y+x+1)^2-xy \qquad x=u^2-v \qquad y=v-1$$