Lab Session - QDCS

Sept 19, 2025

Recall that $\mathcal{H}$ is the Hilbert space generated from $|0\rangle, |1\rangle$.

1 Bell Basis

Consider the ket vectors of $\mathcal{H} \otimes \mathcal{H}$

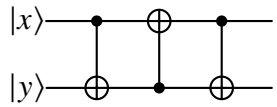
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \qquad |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \qquad |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Questions:

1. Show that these 4 vectors form an orthonormal basis
2. Consider the ket vector
$$|\phi\rangle = \frac{2}{\sqrt{5}}|01\rangle + \frac{i}{\sqrt{5}}|10\rangle$$
   (a) Show that this is a vector of norm 1.
   (b) Write it as a linear combination of $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$ and $|\Psi^-\rangle$.
   (c) Compute $\langle \phi \mid \Psi^+ \rangle$.

2 Reversible Computation

Consider the circuit



What does it compute? Lay out the details in a compact manner (do not use matrices!).

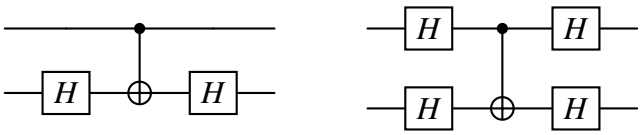
3 Pauli and Hadamard Gates

Compute the following

$$H \cdot X \cdot H, \qquad H \cdot Z \cdot H$$

4 CNOT and Hadamard Gates

Consider the circuits.



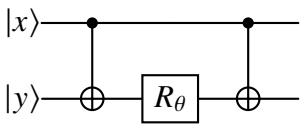
For each of them, give a simpler, equivalent circuit, and the corresponding linear map in function-style and in matrix form. Make sure to give the ordering of basis states you rely on.

5 CNOT and Phase Gate

Remember that the gate R_θ corresponds to the matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

(with θ an angle). Consider the circuit



with θ a real number. What does it compute? Give it in "function" form, and in matrix form (do not forget the basis ordering used for the representation!). Lay out the details in a convincing way.