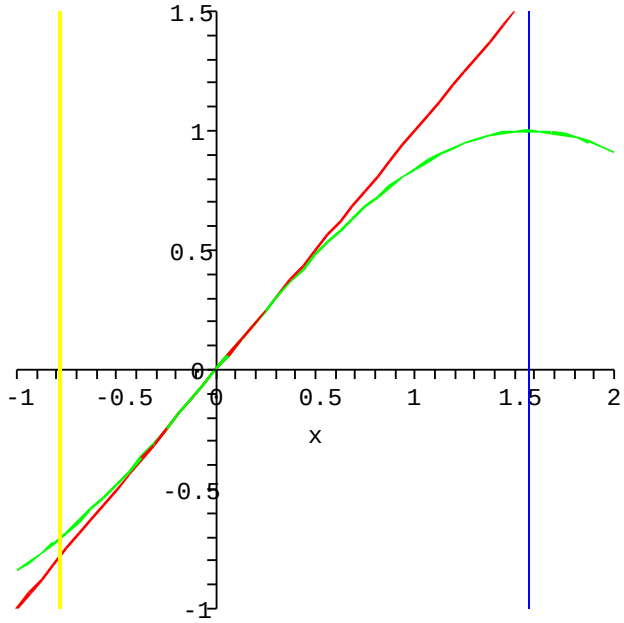
Selected solutions for the suggested exercises, Sections 8.1-2 (and a bit of 5.2)

(1) Find the area between the following curves:

$$(v) \quad y = x \quad y = \sin(x) \quad x = -\tfrac{\pi}{4} \quad x = \tfrac{\pi}{2}.$$

The curves looks like (green is $y = \sin(x)$, red is $y = x$):



The intersection point is 0. Before 0, $\sin(x)$ is above x, after it is below. The area is then:

$$\begin{aligned}
\int_{-\frac{\pi}{4}}^{0} (sin(x) - x)dx + \int_{0}^{\frac{\pi}{2}} (x - \sin(x))dx &= [-\cos(x) - \frac{x^2}{2}]_{-\frac{\pi}{4}}^{0} + [\frac{x^2}{2} + \cos(x)]_{0}^{\frac{\pi}{2}} \\
&= (-\cos(0) - \frac{0}{2}) - (-\cos(-\frac{\pi}{4}) - \frac{(-\frac{\pi}{4})^2}{2}) + (\frac{(\frac{\pi}{2})^2}{2} + \cos(\frac{\pi}{2})) - (\frac{0}{2} + \cos(0)) \\
&= (-1 - 0) - (-\frac{\sqrt{2}}{2} - \frac{\pi^2}{32}) + (\frac{\pi^2}{8} + 0) - (0 + 1) \\
&= \frac{\sqrt{2}}{2} + \frac{5\pi^2}{32} - 2
\end{aligned}$$