

## List of suggested exercises, Section 9.1-4 For the DGD of June 26th and 28th.

### Sequences (Section 9.1)

- (1) For each of the following sequence, write down the 5 first terms, and decide whether it converges or not. If it does, find the limit.

$$(\#22) \quad (-0.3)^n \quad (\#23) \quad 3 + e^{-n} \quad (\#29) \quad \frac{\sin(n)}{n} \quad (\#27) \quad \frac{2n+1}{n} \quad (\#30) \quad \frac{2n + (-1)^n 5}{4n + (-)^n 3}$$

- (2) Find a recursive definition for the sequence:

$$(\#48) \quad 3, 5, 9, 17, 33, \dots \quad (\#50) \quad 1, 3, 6, 10, 15, \dots \quad (\#51) \quad 1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \dots$$

### Series (Sections (9.2-4)

- (1) For each of the following sequence of number, decide which are the first terms of a geometric series and which are not. For those who are, compute the value of the corresponding geometric series, if it exists.

$$\begin{array}{ll} 3 + 12 + 48 + 192 + 768 + \dots & \frac{1}{3} + \frac{1}{2} + \frac{3}{4} + \frac{9}{8} + \frac{27}{16} + \dots \\ \frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \frac{81}{512} + \dots & \frac{1}{3} + \frac{1}{4} + \frac{2}{16} + \frac{9}{64} + \frac{27}{256} + \dots \\ \frac{1}{256} - \frac{1}{160} + \frac{1}{100} - \frac{2}{125} + \frac{16}{625} + \dots & \frac{1}{27} - \frac{1}{63} + \frac{1}{146} - \frac{1}{353} + \frac{3}{2401} + \dots \end{array}$$

- (2) (exercise numbers are from Section 9.3). For each of the following series, decide whether or not they diverge. If they converge, give their value.

$$\begin{array}{lll} (\#10) \quad \sum_{n=0}^{\infty} \frac{3}{n+2} & (\#11) \quad \sum_{n=1}^{\infty} \frac{3}{(2n-1)^2} & (\#16) \quad \sum_{n=1}^{\infty} \left( \left( \frac{3}{4} \right)^n + \frac{1}{n} \right) \\ (\#17) \quad \sum_{n=1}^{\infty} \frac{n+2^n}{n2^n} & (\#18) \quad \sum_{n=1}^{\infty} \frac{\ln(n)}{n} & \end{array}$$

- (3) Find the value of the following series:

$$\begin{array}{lll} \sum_{n=1}^{\infty} \ln \left( \frac{n+1}{n} \right) & \sum_{n=4}^{\infty} \frac{1}{n(n+2)} & \sum_{n=2}^{\infty} \frac{1}{(3n-2)(3n+1)} \\ \sum_{n=1}^{\infty} \ln \left( \frac{n}{2n+5} \right) & \sum_{n=3}^{\infty} \left( \frac{-3}{\pi} \right)^{n-1} & \sum_{n=2}^{\infty} 3^{-n} 8^{n+1} \\ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^{2n}}{2^{3n+1}} & \sum_{n=5}^{\infty} \frac{1}{e^{2n}} & \sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} \quad \sum_{n=1}^{\infty} \left( \frac{-3}{\pi} \right)^{n-1} \\ \sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} & \sum_{n=1}^{\infty} \left( \frac{1}{3^{n+3}} + \frac{1}{2^{2n+1}} \right) & \end{array}$$

(4) Express the following numbers as a series, then as a ratio of integers:

$$0.2\overline{5} = 0.255555\cdots \quad 0.\overline{307} = 0.307307307\cdots \quad 1.\overline{123} = 1.123232323\cdots$$

(5) (Section 9.4) Decide whether the following integrals converges or diverges. State the test you are using. Make sure to verify that the hypotheses are satisfied.

$$\begin{array}{lll} \text{(#5)} \quad \sum_{n=1}^{\infty} \frac{1}{n^4 + e^n} & \text{(#9)} \quad \sum_{n=1}^{\infty} \frac{2^n + 1}{n2^n - 1} & \text{(#11)} \quad \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} \\ \text{(#15)} \quad \sum_{n=1}^{\infty} \frac{2^n}{n^3 + 1} & \text{(#16)} \quad \sum_{n=1}^{\infty} (-1)^n \left(2 - \frac{1}{n}\right) & \text{(#19)} \quad \sum_{n=1}^{\infty} \cos(n\pi) \\ \sum_{n=1}^{\infty} \frac{2n^4 + n^3 - n + 4}{\sqrt[3]{5n^{17} - 2n^3 + 6}} & \text{(#30)} \quad \sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} - \sqrt{n+2}} & \text{(#31)} \quad \sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n}\right) \\ \text{(#48)} \quad \sum_{n=1}^{\infty} \frac{\sin(x)}{n^2} & \text{(#51)} \quad \sum_{n=2}^{\infty} \frac{3}{\ln(n^2)} & \text{(#53)} \quad \sum_{n=1}^{\infty} \frac{n(n+1)}{\sqrt{n^3 + 2n^2}} \end{array}$$

(6) Approximate the following series within 0.001.

$$\text{(#54)} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \quad \text{(#55)} \quad \sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^{n-1} \quad \text{(#56)} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n)!}$$

(7) Determine if the following series are absolutely convergent, conditionally convergent or divergent.

$$\begin{array}{lll} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}} & \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} & \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1} \\ \sum_{n=1}^{\infty} \frac{(-2)^n}{n3^{n+1}} & \sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2} & \sum_{n=1}^{\infty} \frac{(-1)^n \arctan(n)}{n^3} \\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}5^{n-1}}{(n+1)^24^{n+2}} & \sum_{n=1}^{\infty} \frac{(-2)^n n^2}{(n+2)!} & \sum_{n=1}^{\infty} \left(\frac{1-3n}{1+4n}\right)^n \\ \sum_{n=1}^{\infty} \frac{(-n)^n}{5^{2n+3}} & \sum_{n=1}^{\infty} \frac{n!}{(-n)^n} & \sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{6}\right)}{n\sqrt{n}} \end{array}$$