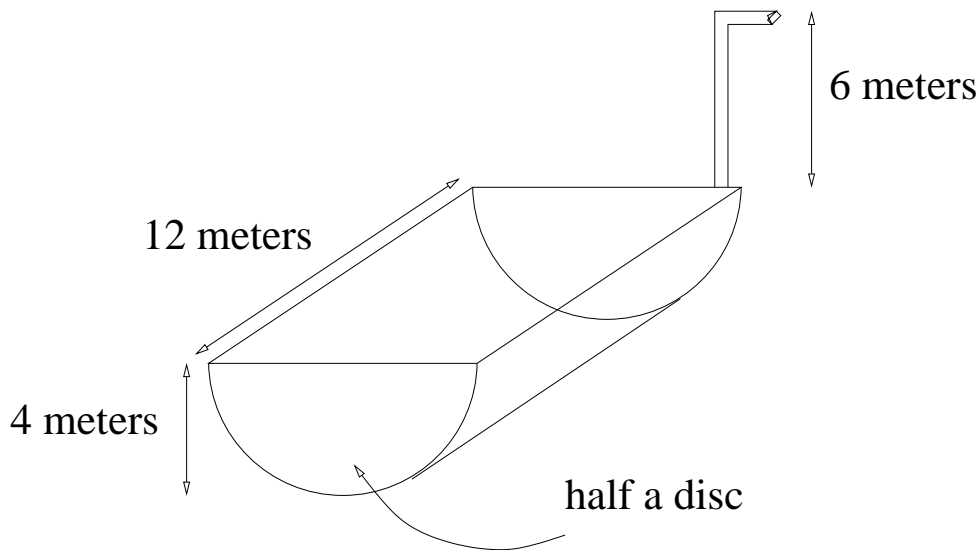# Mat 1322 3X – Summer 2007 – Homework #3.

To hand in Tuesday, June 19th.

Question 1. (5 points) A tank containing water has the form of half a cylinder on its side, has radius 4 meters and length 12 meters. If the tank is filled with water (density 1000 kg/m^3), what is the work done when emptying the tank from a point 6 meters above the side ?

6 meters

12 meters

4 meters

half a disc

Question 2. (3 points) Consider the following improper integral:

$$\int_0^\infty x^2 e^{-2x} dx$$

(a) Write the definition of this integral.

(b) Decide whether or not it converge. If it does, compute its value.

Question 3. (2 points) Decide whether the following integral converges or diverges using the comparison test:

$$\int_0^1 \frac{dx}{\sqrt[5]{x^6 + x^3}}$$

Question 4. (5 points each) Consider the following initial value problems:

$$
\begin{array}{lll}
(i) & \frac{dy}{dx} e^x = 3y^2 & y(0) = 1 \\
(ii) & \frac{y'}{y} = x\cos(x) & y(0) = e
\end{array}
$$

1

Note: for the equation (ii), x is in radian (so for example, $\cos(\pi) = -1$).

For each of them:

(a) Find the exact solution for this initial value problem.

(b) Use the Euler method with $\Delta x = 0.1$ to estimate the value of $y(0.4)$.

(c) Compute the error between the approximated solution of (b) and the exact value from (a).

Question 5. (5 points) A murder has been committed in an hotel room. The body is found at midnight and is 29°C. The room is 20°C, constant. Two hours later, the temperature of the body is 24°C.

Write down the differential equation satisfied by the temperature of the body, assuming it follows Newton's cooling law. Solve the equation, and sketch the graph of the temperature in function of time. Knowing that the normal temperature of a body is 37°C, when was the murder committed ?

Question 6. (5 points) (#10 p.565) The population of a species of elk on Reading Island in Canada has been monitored for some years. When the population was 600, the relative birth rate was 35% and the relative death rate was 15%. As the population grew to 800, the corresponding figures were 30% and 20%. The island is isolated so the is no hunting or migration.

Note: the relative growth rate is computed by

$$(\text{relative birth rate}) - (\text{relative death rate}).$$

(a) Write a differential equation to model the population as a function of time. Assume that relative growth rate is a linear function of population.

(b) What is the equilibrium size of the population ? Today, there are 900 elk on the island. How do you expect the population to change in the future ?

(c) Oil has been discovered on a neighborhood island and the oil companies want to move 450 elk of the same species to Reading Island. What effect would this move have on the elk population in the future ?

(d) Assuming the elk are moved to Reading Island, sketch the population on Reading Island as a function of time. Start before the elk are transferred and continue for some time into the future.

Question 7. (5 points) A tank of volume $4 \cdot 10^6$ liters contains initially 5kg of salt. Water with salt concentration 0.2kg/L is flowing in at a rate of 30L/min. Water with salt concentration 0.5kg/L is flowing in at a rate of 70L/min. The water is well mixed and flow out of the tank at a rate of 100L/min.

(a) Write down the differential equation satisfied by $Q(t)$ the quantity of salt in the tank at time t (in minute).

(b) Solve the differential equation.

(c) What is the quantity of salt in the tank after a long time ?

(d) Sketch the graph of the quantity of salt in the tank in term of t.