Mat 1322 3X – Summer 2007 – Homework #4.

To hand in Thursday, June 28th.

Question 1. (4 points) A fish population $P(t)$ in an artificial lake follows the equation:

$$\frac{dP}{dt} = 0.2P - 0.0004P^2$$

where t is measured in year since 2000, year when 75 fishes were introduced.

(1) Sketch the graph of the population in term of the time.

(2) What is the number of fishes in the lake in 2030 ?

(3) When will the lake contain 1250 fishes ?

Question 2. (6 points) On Ben's Island, a certain species of bird lives in relative isolation. When there were 200 birds on the island, the relative birth rate was 50% and the relative rate of death was 10%. When the number of birds was 700, the relative birth rate was 40% and the relative death rate was 20%.

(1) Assuming the relative rate of growth is linear in function of the population, write down the equation satisfied by by P.

(2) If the initial population is 300, sketch the graph of the population in term of the time.

(3) When the population reaches 1100, the elk from Reading Island, finding their island crowded, send 500 birds of the same species on Ben's Island. Sketch the graph of the population indicating the moment when the new birds arrive.

Question 3. (8 points) For each of the following series, decide whether they converge or not. If they do, find their limit.

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{2n^4+3}}$

(2) $\displaystyle 8 - 6 + \frac{9}{2} - \frac{27}{8} + ...$

(3) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n + 2^{2n+1}}{8^n}$

(4) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+5)}$

Question 4. (8 points) For each of the following series, decide whether they converge or not using the integral test or the (limit) comparison test.

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin^2 n}{n^{4/3}}$

(2) $\displaystyle\sum_{n=1}^{\infty} n^3 e^{-3n^4}$

(3) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{\sqrt[4]{n^9+n^3}}$

(4) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{7^n+5}$